Comparison of Pixel Correlation Induced by Space-Filling Curves on 2D Image Data

Stéphane Duguay & Steven Pigeon

Canada
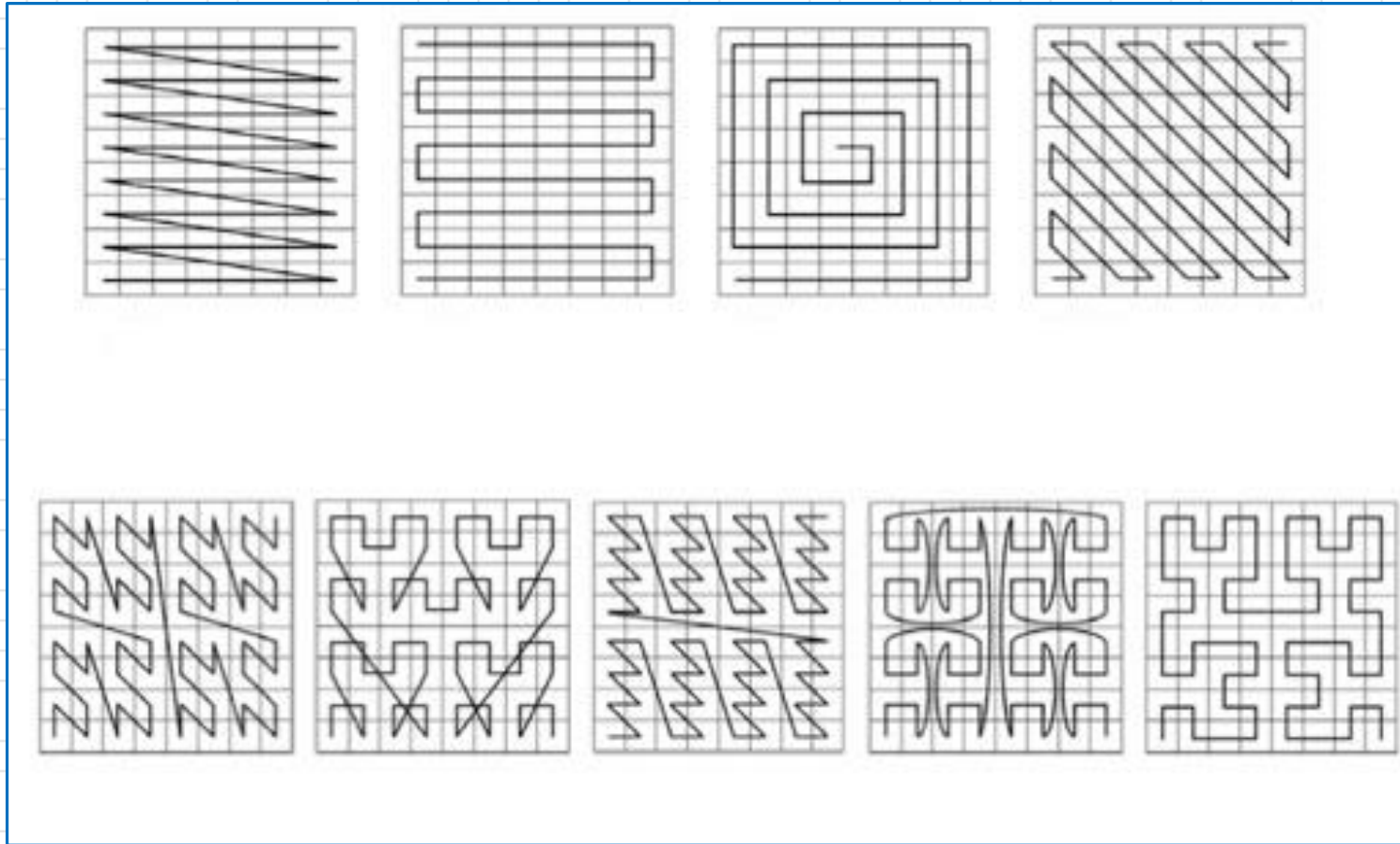
UQAR
Université du Québec
à Rimouski

What are Space-Filling Curves?

A *space-filling curve* (SFC) is a curve that allows mapping from a multi-dimensional space to a one-dimensional space.

It is a path that passes through every space unit in the multi-dimensional space so that every space unit is visited exactly once.

Some Space-Filling Curves

Why SFCs for image compression?

In natural images, neighboring pixel values are usually highly correlated.

SFCs preserve pixel locality when they are used as paths to traverse 2D image data.

SFCs, such as Hilbert's, Peano's, or data-dependent adaptive SFCs, are used for dimensionality reduction in a way that preserves inter-pixel correlation. That property is then exploited in prediction- and transform-based compression algorithms.

Problem statement

We investigated the distribution of pixel values correlation induced by data-independent SFCs, and which SFCs provide the best possible pixel values correlation for <u>natural photographic</u> image traversal.

Methodology overview

To quantify the pixel values correlation property of space-filling curves, we accumulated the error energy for all possible Hamiltonian paths on each pixel subsets (patches) of different sizes for **100 natural photographic images.**

Methodology: resolutions and color planes

- Each 24-bit image in the test set was processed at three different resolutions:

  - 5202×3465 (original)

  - 2601×1733 (half)

  - 1300×866 (quarter).

  - Down-sampling was performed using ImageMagick's Blackman filter.

- The test set therefore comprises 300 images, for a total of **900 8-bit image planes.**

Methodology: patches

- In our work, a patch is a n×n pixel region of an image, comprising all color planes.

- Because the number of possible paths grows very rapidly, we considered patches of sizes 4×4 to 6×6.

- To generate a great number of patches and to guard against alignment effects, all distinct **n×n** patches were extracted from an image, for a total of **(x − n + 1)×(y − n + 1)** patches for a **x×y** pixel image.

Methodology: number of patches

- **Number of 4x4 8-bit patches:**

(300 * (5202-4)*(3465-4)) + (300 * (2601-4)*(1733-4))

   + (300 * (1300-4)*(866-4)) = 7 079 292 900

- **Number of 5x5 8-bit patches:**

(300 * (5202-5)*(3465-5)) + (300 * (2601-5)*(1733-5))

   + (300 * (1300-5)*(866-5)) = 7 074 750 900


- **Number of 6x6 8-bit patches:**

(300 * (5202-6)*(3465-6)) + (300 * (2601-6)*(1733-6))

   + (300 * (1300-6)*(866-6)) = 7 070 210 700

- **Total number of patches = 21 224 254 500 = 2.12242545 × 10^{10}**

Methodology: Hamiltonian paths

- A Hamiltonian path on a grid graph is a path that visits every node exactly once.

- For our application, a node is a pixel, and the connectivity of the graph is limited by N-S-E-W neighbors within the n×n pixel patch.

- We intended to find the distribution of energy induced by <u>all possible directed Hamiltonian paths</u> in a n×n grid graph.

(a) Hilbert

(b) Row-prime order

(c) Spiral

(d) One of many others
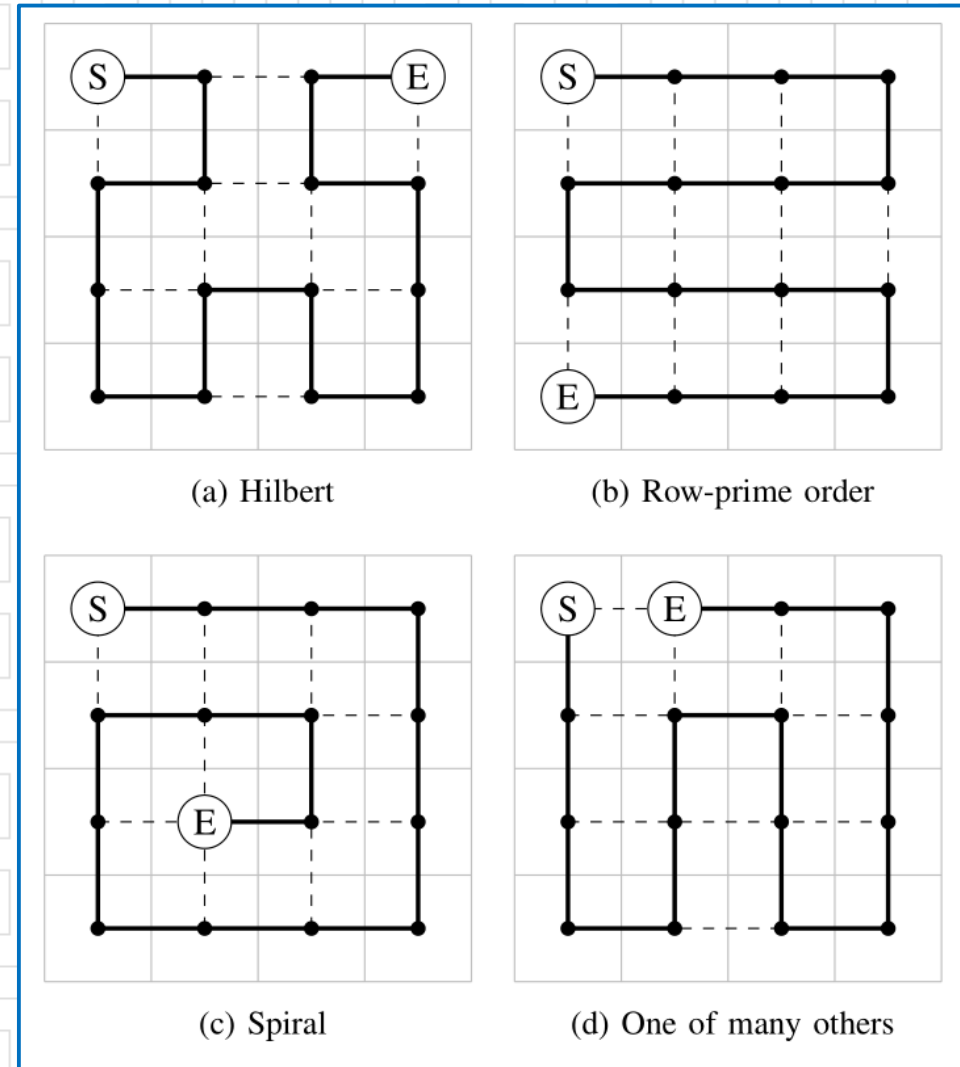
Methodology: energy induced estimation

- Classic approach: the sum of the squared difference between pixels along a path.

- The energy for a **n×n** single color component patch **c** unwound by path **h** is given by:

$$\mathcal{E}(h,c) = \sum_{j=0}^{n^2-2} \left(c_{h_{j+1}} - c_{h_j}\right)^2$$

- We then accumulated the energy of every patch **p** of every color components **c** of every image **i** in the image test set **I**. The total energy over the test set is given by:

$$\mathcal{E}(h,I) = \sum_{i \in I} \sum_{p \in i} \sum_{c \in p} \mathcal{E}(h,c)$$

Normalization

The accumulated energies are normalized between zero and one, with zero corresponding to the lowest energy observed for a given experiment, and one to the highest. The histogram is then normalized in height so that the surface is one—a probability distribution.
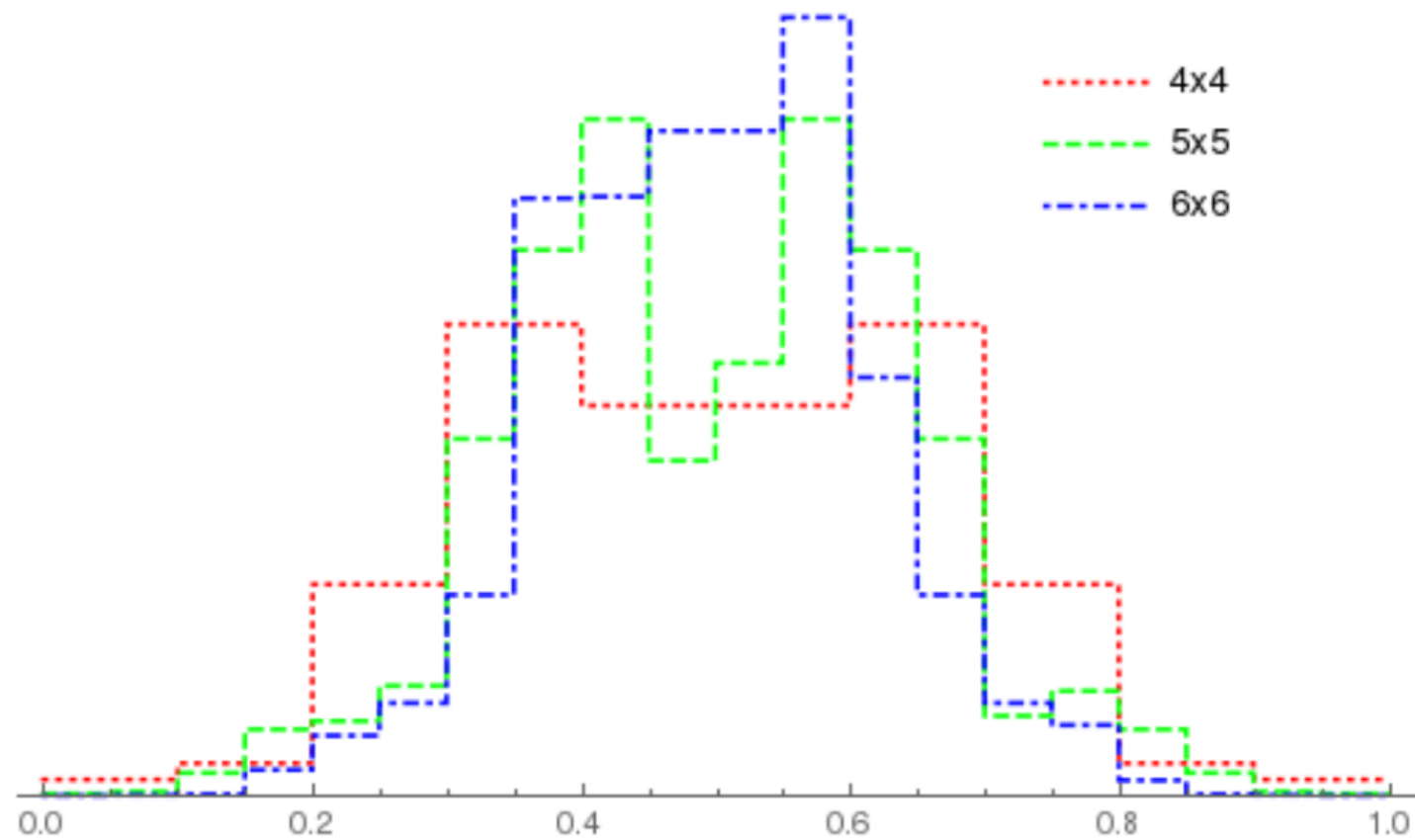
Results: distributions

Figure 3. The distribution of energies induced by paths over 4×4 ■, 5×5 ■, and 6×6 ■ patches, on full resolution images.
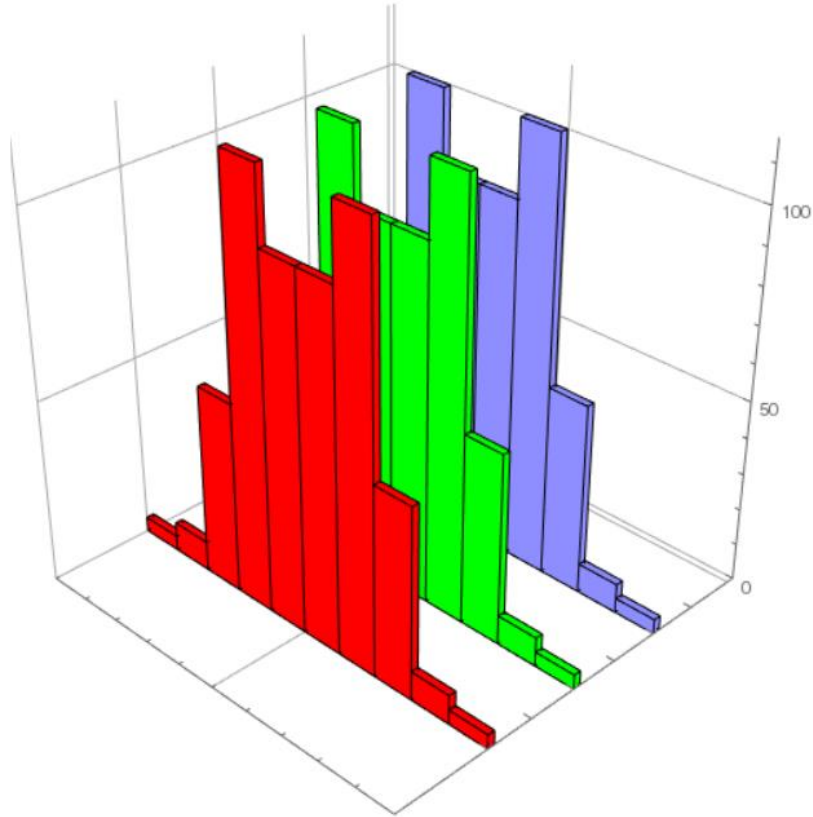
Results: distributions

Figure 4. The distribution of energies induced by paths over 4×4 patches, from quarter ■, half ■, and full resolution ■ images.
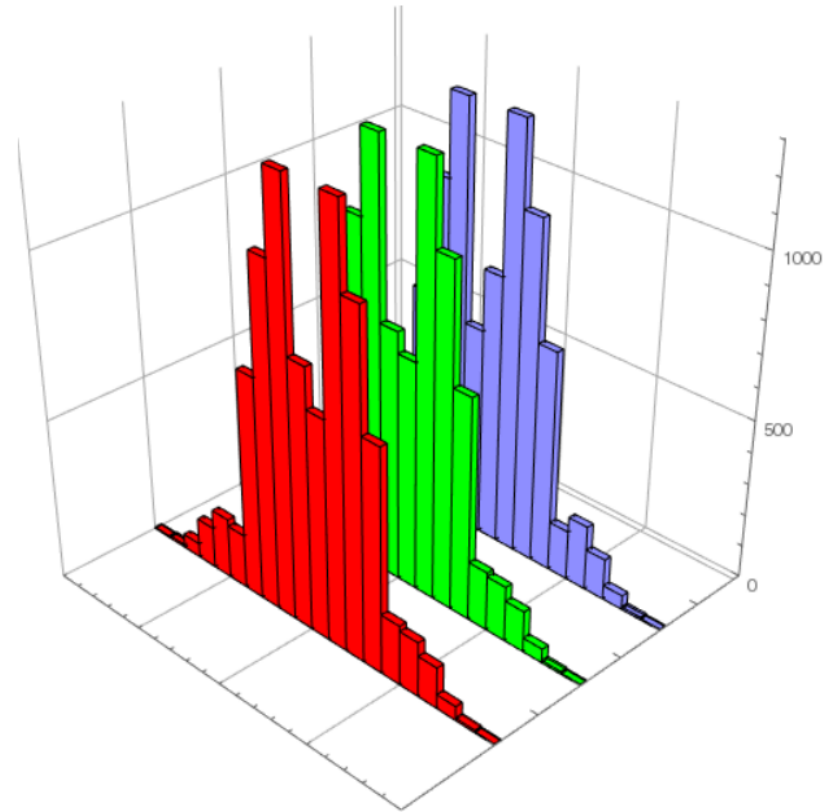
4x4 patches

Figure 5. The distribution of energies induced by paths over 5×5 patches, from quarter ■, half ■, and full resolution ■ images.

5x5 patches
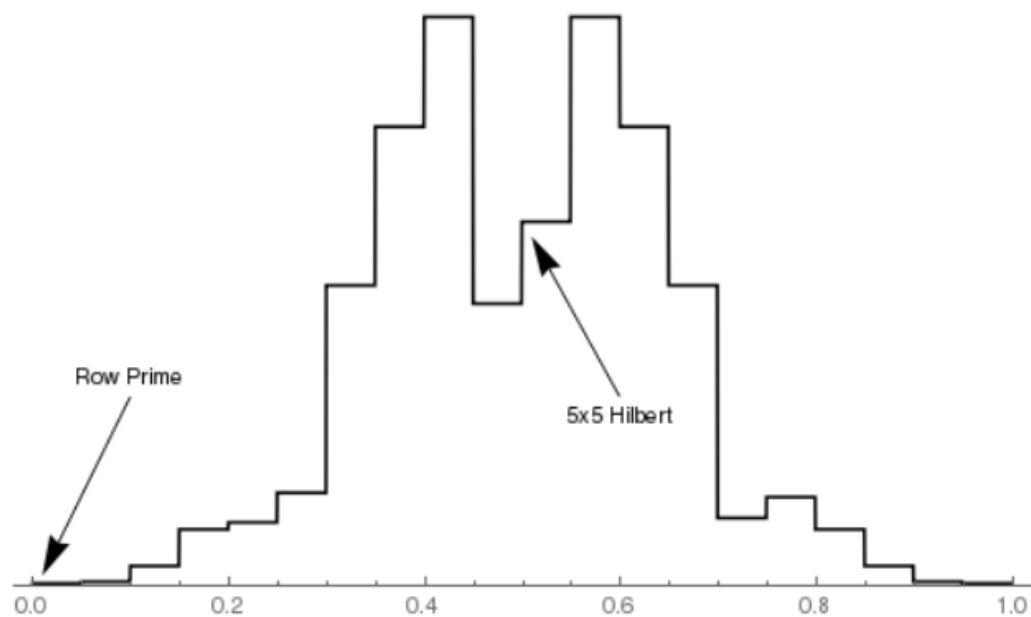
Discussion

Figure 6. The location of various curves on 5×5 patches.
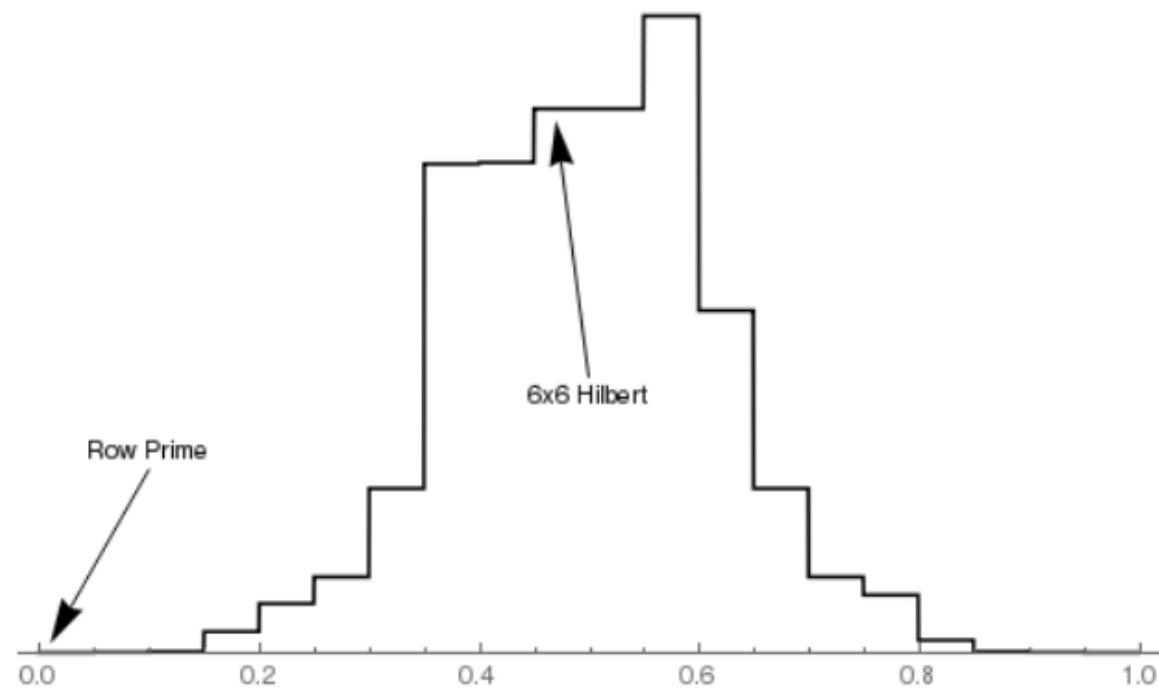

Figure 7. The location of various curves on 6×6 patches.

Discussion

The results show that, contrary to conventional wisdom, neither Hilbert's nor Peano's curve minimize decorrelated energy on (natural) images: they perform as the average SFC.

It seems that they merely exploit locality, or "spatial correlation", which also may mean crossing an edge many times.

Discussion

One may question if the effects observed are an artifact of the data set used. While we have yet to run the test with all the curves on larger data sets, we have measured that the vertical energy (column-prime) and horizontal energy (row-prime) shows approximately the same relative difference and preference to row-prime order.

Table II. COMPARISON OF AVERAGE HORIZONTAL AND VERTICAL ENERGIES PER PIXEL

Data Set (count)	Average Energy Per Pixel		
	Horiz.	Vert.	Relative Diff.
Kodak True Color (24)	23.62	26.06	9.36%
Our Data Set (100)	33.67	34.30	1.84%
Flickr Faces (70 000)	31.13	27.77	-10.79%
Open Images Ex (478 215)	122.78	124.34	1.25%

Next steps

- Expand the experiment to include bigger patches (up to 8x8) and bigger data sets.

- Put together a theoretical framework explaining our results using a locality measurement(s) or metric(s) for SFCs.

- All figures hint that energy is somewhat normally distributed if we consider all paths—we may even conjecture that as the patch size grows, the distribution (over all SFC) will converge to a binomial distribution.

Questions?